Analysis of Coupled Inset Dielectric Guide Structure

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***Abstract*—The propagation and coupling characteristics of the inset dielectric guide coupler are theoretically considered in this paper. Rigorous solutions for the dispersion relation and the coupling coefficient are presented in a rapidly convergent series. Numerical computations illustrate the behaviors of dispersion, coupling, and field distribution in terms of frequency and coupler geometry.**

***Index Terms*—Coupling, dispersion, Fourier transform, IDG.**

I. INTRODUCTION

THE inset dielectric guide (IDG) is a dielectric-filled rectangular groove guide, and its guiding characteristics are well known [1], [2]. The coupled IDG consisting of a double IDG has also been extensively studied to assess its utility as a coupler [3]. The IDG coupler may be used for a practical directional coupler and bandpass filter due to power splitting and filter characteristics with low radiation loss and simple fabrication. It is of theoretical and practical interest to consider the wave propagation characteristics of an IDG coupler, which consists of a multiple number of parallel IDG. We use the Fourier transform and mode matching as used in a single IDG analysis [2], thus obtaining a rigorous solution for the IDG coupler in a rapidly convergent series.

II. FIELD ANALYSIS

Consider an IDG coupler with a conductor cover in Fig. 1 (N: the number of IDG). The effect of a conductor cover at $y = b$ is negligible on dispersion when b is greater than one-half wavelength [2]. Assume the hybrid mode propagates along the z-direction, such as $\overline{H}_z(x, y, z) = H_z(x, y)e^{i\beta z}$ and $\overline{E}_z(x, y, z) = E_z(x, y)e^{i\beta z}$ with $e^{-i\omega t}$ time-factor omission. In regions I ($-d < y < 0$) and II ($0 < y < b$), the field components are

$$E_z^I(x, y) = \sum_{n=0}^{N-1} \sum_{m=0}^{\infty} p_m^n \sin a_m(x - nT) \sin \xi_m(y + d) \cdot \left[u(x - nT) - u(x - nT - a)\right] \quad (1)$$

$$H_z^I(x, y) = \sum_{n=0}^{N-1} \sum_{m=0}^{\infty} q_m^n \cos a_m(x - nT) \cos \xi_m(y + d) \cdot \left[u(x - nT) - u(x - nT - a)\right] \quad (2)$$

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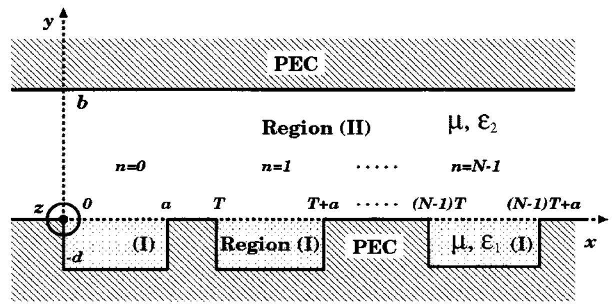

Fig. 1. Geometry of the IDG coupler.

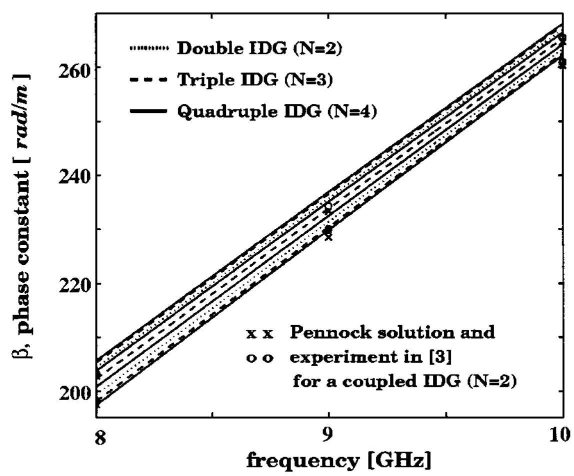

Fig. 2. Dispersion characteristics of the IDG coupler filled with (Teflon) PTFE ($\epsilon_r = 2.08$) for $a = 10.16$ mm, $d = 15.24$ mm, $T = 11.86$ mm, $b = 15$ mm, and $N = 2, 3, 4$.

$$E_z^{II}(x, y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\tilde{E}_z^+ e^{i\eta y} + \tilde{E}_z^- e^{-i\eta y}\right] e^{-i\zeta x} \, d\zeta \quad (3)$$

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where

$$a_m = m\pi/a$$
$$\xi_m = \sqrt{k_1^2 - a_m^2 - \beta^2}$$
$$\eta = \sqrt{k_2^2 - \zeta^2 - \beta^2}$$
$$k_1 = \omega\sqrt{\mu\epsilon_1}$$
$$k_2 = \omega\sqrt{\mu\epsilon_2} \quad \text{and} \quad u(\cdot)$$

is a unit step function. To determine the modal coefficients p_m^n and q_m^n, we enforce the boundary conditions on the E_x-, E_z-,

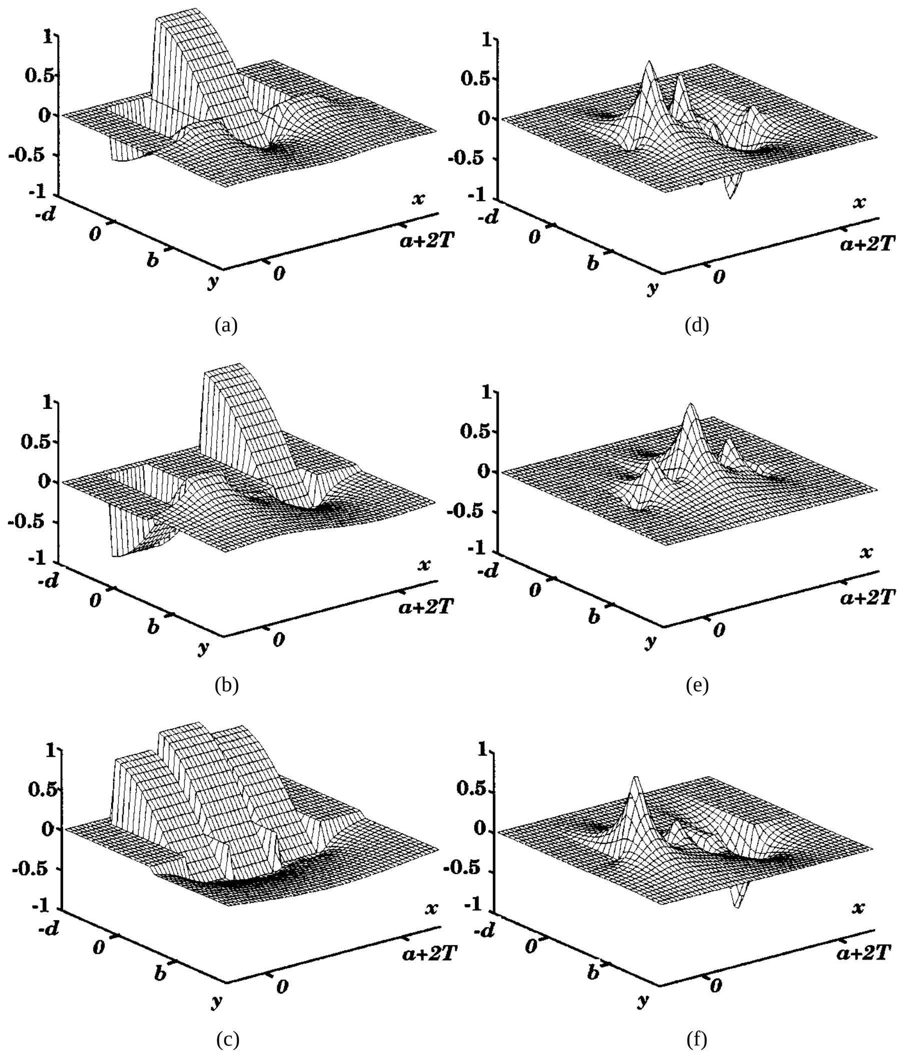

Fig. 3. Field distributions for: (a) HE_{31}, (b) HE_{21}, (c) HE_{11} (H_z-field), and (d) HE_{31}, (e) HE_{21}, (f) HE_{11} (E_z-field) when $N = 3$.

H_x-, and H_z-field continuities. Applying a similar method as was done in [2], we obtain

$$\sum_{n=0}^{N-1}\sum_{m=0}^{\infty}\Big\{p_m^n A_m I_{ml}^{np} - q_m^n \cdot \Big[B_m I_{ml}^{np} + \frac{a}{2}\cos(\xi_m d)\,\delta_{ml}\,\delta_{np}\alpha_m\Big]\Big\} = 0, \quad l = 0, 1, \ldots \tag{5}$$

$$\sum_{n=0}^{N-1}\sum_{m=0}^{\infty}\Big\{p_m^n\Big[\sin(\xi_m d) J_{ml}^{np} + \frac{a}{2}C_m\,\delta_{ml}\,\delta_{np}\Big] + q_m^n\frac{a}{2}D_m\,\delta_{ml}\,\delta_{np}\Big\} = 0 \tag{6}$$

where δ_{ml} is the Kronecker delta, $\alpha_0 = 2$, $\alpha_m = 1 (m = 1, 2, \ldots)$

$$A_m = \frac{k_2^2 - k_1^2}{k_1^2 - \beta^2}\,\frac{\beta a_m}{\omega\mu}\,\sin(\xi_m d) \tag{7}$$

$$B_m = \frac{k_2^2 - \beta^2}{k_1^2 - \beta^2}\,\xi_m \sin(\xi_m d) \tag{8}$$

$$C_m = \frac{k_2^2 - \beta^2}{k_1^2 - \beta^2}\,\frac{\epsilon_1}{\epsilon_2}\,\xi_m \cos(\xi_m d) \tag{9}$$

$$D_m = \frac{k_2^2 - k_1^2}{k_1^2 - \beta^2}\,\frac{\beta a_m}{\omega\epsilon_2}\,\cos(\xi_m d) \tag{10}$$

$$I_{ml}^{np} = \frac{a}{2}\,\frac{\alpha_m\,\delta_{ml}\,\delta_{np}}{\eta_m\tan(\eta_m b)} - \frac{i}{b}\sum_{v=0}^{\infty}\frac{\zeta_v f(\zeta_v)}{\alpha_v\big(\zeta_v^2 - a_m^2\big)\big(\zeta_v^2 - a_l^2\big)} \tag{11}$$

$$J_{ml}^{np} = \frac{a}{2}\,\frac{\eta_m\,\delta_{ml}\,\delta_{np}}{\tan(\eta_m b)} - \frac{a_m a_l}{b}\,i\sum_{v=1}^{\infty}\frac{\left(\frac{v\pi}{b}\right)^2 f(\zeta_v)}{\zeta_v\big(\zeta_v^2 - a_m^2\big)\big(\zeta_v^2 - a_l^2\big)} \tag{12}$$

$$f(\zeta) = \Big((-1)^{m+l} + 1\Big)e^{i\zeta_v|n-p|T} - (-1)^m e^{i\zeta_v|(n-p)T+a|} - (-1)^l e^{i\zeta_v|(n-p)T-a|} \tag{13}$$

$$\zeta_v = \sqrt{k_2^2 - (v\pi/b)^2 - \beta^2}$$

$$\eta_m = \sqrt{k_2^2 - (m\pi/a)^2 - \beta^2}.$$

A dispersion relationship may be obtained by solving (5) and (6) for β as follows:

$$\begin{vmatrix} \Psi_1 & \Psi_2 \\ \Psi_3 & \Psi_4 \end{vmatrix} = 0 \tag{14}$$

where the elements of Ψ_1, Ψ_2, Ψ_3, and Ψ_4 are

$$\begin{aligned} \psi_{1,ml}^{np} &= A_m I_{ml}^{np} \\ \psi_{2,ml}^{np} &= -B_m I_{ml}^{np} - \frac{a}{2}\cos(\xi_m d)\,\delta_{ml}\,\delta_{np}\alpha_m \\ \psi_{3,ml}^{np} &= \sin(\xi_m d) J_{ml}^{np} + \frac{a}{2} C_m\,\delta_{ml}\,\delta_{np} \\ \psi_{4,ml}^{np} &= \frac{a}{2} D_m\,\delta_{ml}\,\delta_{np}. \end{aligned} \tag{15}$$

When $N = 1$ (a single IDG case), (14) reduces to [2, eq. (30)]. When $\epsilon_1 = \epsilon_2$, (14) results in the dispersion relationship for the rectangular groove guide in [4] and [5] as

$$|\Psi_2|\,|\Psi_3| = 0 \tag{16}$$

where Ψ_2 and Ψ_3, respectively, represent TE and TM modes in the groove guide with an electric wall placed at $y = b$. In a dominant-mode approximation ($m = 0$, $l = 0$), (14) reduces to

$$|\Psi_2| = 0. \tag{17}$$

When $N = 2$, (17) yields a simple dispersion relation as

$$\psi_{2,00}^{00} = \pm\psi_{2,00}^{10} \tag{18}$$

where each $\pm$ sign corresponds to the odd and even modes in [3]. To calculate the coupling coefficients between the guides, we introduce the eigenvector X^s associated with the eigenvalue $\beta = \beta_s$ ($s = 1, \ldots, N$), where the elements of X^s are $[x_0, x_1, \ldots, x_{N-1}]^T$ and $x_n = H_z^I(nT, -d) = \sum_{m=0}^{\infty} q_m^n$. Note that q_m^n is obtained by solving (5) and (6) with $\beta = \beta_s$ determined by (14). The field at z, $h_z^n(z) \triangleq \overline{H}_z^I(nT, -d, z)$ in the nth IDG, is related to the field at $z = 0$ through a transformation with the eigenvector [6]

$$\begin{bmatrix} h_z^0(z) \\ \vdots \\ h_z^{N-1}(z) \end{bmatrix} = [\tilde{X}^1 \cdots \tilde{X}^N] \begin{bmatrix} e^{i\beta_1 z} & \cdots & 0 \\ \vdots & \vdots & \vdots \\ 0 & \cdots & e^{i\beta_N z} \end{bmatrix} [\tilde{X}^1 \cdots \tilde{X}^N]^T \begin{bmatrix} h_z^0(0) \\ \vdots \\ h_z^{N-1}(0) \end{bmatrix} \tag{19}$$

where $\tilde{X}^s = X^s/\|X^s\|$ is the normalized eigenvector and $(\cdot)^T$ denotes the transpose of $(\cdot)$. We define the coupling coefficient at $z = L$ between the ith and jth guides as

$$C_{ij} = 20\log_{10}\left|h_z^i(L)\right| \tag{20}$$

where $h_z^p(0) = \delta_{pj}$. For instance, $N = 2$, $h_z^0(0) = 1$, and $h_z^1(0) = 0$, (20) reduces to the coupling coefficient in [3, eq. (39)]. Applying a dominant mode approximation for $N = 2$ and 3, we obtain eigenvectors as

$$\begin{aligned} X^1 &= [1 \quad -1]_{\beta=\beta_1}^T \\ X^2 &= [1 \quad 1]_{\beta=\beta_2}^T, \qquad N = 2 \end{aligned} \tag{21}$$

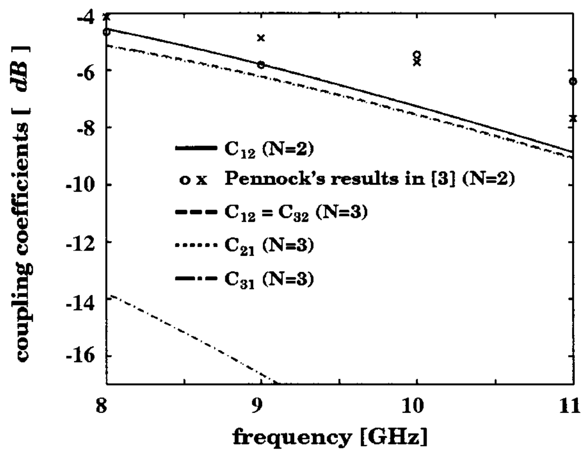

Fig. 4. Behavior of the coupling coefficients C_{ij} where PTFE ($\epsilon_r = 2.08$), $a = 10.16$ mm, $d = 15.24$ mm, $T = 11.86$ mm, $b = 15$ mm, $L = 250$ mm, and $N = 2, 3$.

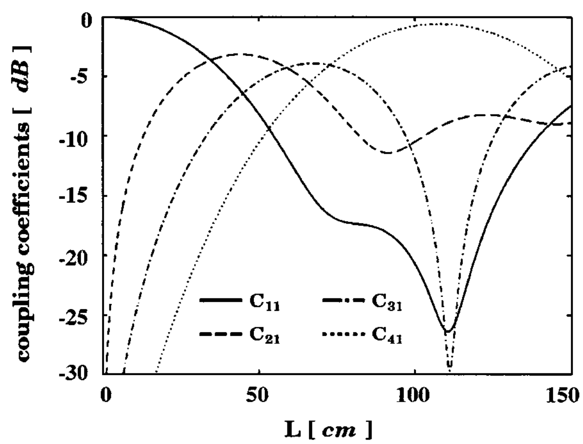

Fig. 5. Behavior of the coupling coefficients C_{ij} versus $z = L$ where PTFE ($\epsilon_r = 2.08$), frequency = 8 GHz, $a = 10.16$ mm, $d = 15.24$ mm, $T = 11.86$ mm, $b = 15$ mm, and $N = 4$.

$$\begin{aligned} X^{1,3} &= \left[\psi_{2,00}^{00} \quad -2\psi_{2,00}^{10} \quad \psi_{2,00}^{00}\right]_{\beta=\beta_1,\beta_3}^T \\ X^2 &= [1 \quad 0 \quad -1]_{\beta=\beta_2}^T, \qquad N = 3. \end{aligned} \tag{22}$$

Fig. 2 illustrates the dispersion characteristics for IDG couplers ($N = 2, 3, 4$), confirming that our solution for $N = 2$ agrees with those in [3] within 1% error. Note that an increase in the number of IDG causes an increase in possible propagating modes. Fig. 3 shows the magnitude plots of H_z and E_z components for three fundamental HE_{p1} modes ($p = 1, 2, 3$), where the subscripts p, and 1 denote the number of half-wave variations of the H_z component in the x- and y-directions, respectively. The field plots illustrate that H_z remains almost uniform in the x-direction within the groove, thus confirming the validity of a dominant-mode approximation in all cases considered in this paper. In Fig. 4, we compare the behavior of the coupling coefficients for $N = 2$ and 3 versus frequency. When $N = 2$, we calculate the coupling coefficient (20) using the measured ($\circ$) and calculated ($\times$) propagation constants in [3]. Note that our theoretical calculation agrees well with the results based on [3]. When $N = 3$, the couplings between adjacent grooves are almost the same ($C_{12} \approx C_{21}$), while the coupling to the far groove is far less ($C_{31} \ll C_{21}$). Fig. 5 illustrates the coupling coefficients versus $z = L$ for $N = 4$. As the excitation wave propagates in the first groove $(n = 0)$, it is coupled to the adjacent grooves $(n = 1, 2, 3)$ in an ordered manner. Note that a maximum power transfer occurs from the first guide to the adjacent ones at $L = 44.0$ cm, 67.3 cm, and 108 cm, successively.

III. CONCLUSION

A simple, exact, and rigorous solution for the IDG coupler has been presented and its dispersion and coupling coefficients have been evaluated. Numerical computations illustrate the field distributions and coupling mechanism amongst the guides. A dominant-mode approximation is shown to be accurate and useful for IDG coupler analysis.

REFERENCES

[1] T. Rozzi and S. J. Hedges, "Rigorous analysis and network modeling of the inset dielectric guide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-35, pp. 823–833, Sept. 1987.

[2] J. K. Park and H. J. Eom, "Fourier-transform analysis of inset dielectric guide with a conductor cover," *Microwave Opt. Technol. Lett.*, vol. 14, no. 6, pp. 324–327, Apr. 1997.

[3] S. R. Pennock, D. M. Boskovic, and T. Rozzi, "Analysis of coupled inset dielectric guides under LSE and LSM polarization," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 916–924, May 1992.

[4] B. T. Lee, J. W. Lee, H. J. Eom, and S. Y. Shin, "Fourier-transform analysis for rectangular groove guide," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 2162–2165, Sept. 1995.

[5] H. J. Eom and Y. H. Cho, "Analysis of multiple groove guide," *Electron. Lett.*, vol. 35, no. 20, pp. 1749–1751, Sept. 1999.

[6] A. Yariv, *Optical Electronics in Modern Communications*. New York: Oxford Book, 1997, pp. 526–531.

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